General Certificate of Education
Advanced Level Examination
June 2010

Mathematics MFP4

Unit Further Pure 4

Tuesday 15 June 2010 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The position vectors of the points P, Q and R are, respectively,

$$\mathbf{p} = \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \quad \text{and} \quad \mathbf{r} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$$

(a) Show that $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are linearly dependent. *(2 marks)*

(b) Determine the area of triangle PQR. *(4 marks)*

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2 Let $\mathbf{A} = \begin{bmatrix} 1 & x \\ 2 & 3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 4-4x & 8 \\ 8x-4 & 4 \end{bmatrix}$.

(a) Find $\mathbf{AB}$ in terms of x. *(2 marks)*

(b) Show that $\mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}} = \mathbf{C}$ for some value of x. *(5 marks)*

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3 The plane Π_1 is perpendicular to the vector $9\mathbf{i} - 8\mathbf{j} + 72\mathbf{k}$ and passes through the point $A(2, 10, 1)$.

(a) Find, in the form $\mathbf{r}\,.\,\mathbf{n} = d$, a vector equation for Π_1. *(3 marks)*

(b) Determine the exact value of the cosine of the acute angle between Π_1 and the plane Π_2 with equation $\mathbf{r}\,.\,(\mathbf{i} + \mathbf{j} + \mathbf{k}) = 11$. *(4 marks)*

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4 The fixed points A and B and the variable point C have position vectors

$$\mathbf{a} = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} \quad \text{and} \quad \mathbf{c} = \begin{bmatrix} 2-t \\ t \\ 5 \end{bmatrix}$$

respectively, relative to the origin O, where t is a scalar parameter.

(a) Find an equation of the line AB in the form $(\mathbf{r} - \mathbf{u}) \times \mathbf{v} = \mathbf{0}$. *(3 marks)*

(b) Determine $\mathbf{b} \times \mathbf{c}$ in terms of t. *(4 marks)*

(c) (i) Show that $\mathbf{a}\,.\,(\mathbf{b} \times \mathbf{c})$ is constant for all values of t, and state the value of this constant. *(2 marks)*

(ii) Write down a geometrical conclusion that can be deduced from the answer to part **(c)(i)**. *(1 mark)*

5 Factorise fully the determinant $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix}$. *(8 marks)*

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6 The line L and the plane Π have vector equations

$$\mathbf{r} = \begin{bmatrix} 7 \\ 8 \\ 50 \end{bmatrix} + t\begin{bmatrix} 6 \\ 2 \\ -9 \end{bmatrix} \quad \text{and} \quad \mathbf{r} = \begin{bmatrix} -2 \\ 0 \\ -25 \end{bmatrix} + \lambda\begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix} + \mu\begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix}$$

respectively.

(a) (i) Find direction cosines for L. *(2 marks)*

(ii) Show that L is perpendicular to Π. *(3 marks)*

(b) For the system of equations

$$\begin{aligned} 6p + 5q + r &= 9 \\ 2p + 3q + 6r &= 8 \\ -9p + 4q + 2r &= 75 \end{aligned}$$

form a pair of equations in p and q only, and hence find the unique solution of this system of equations. *(5 marks)*

(c) It is given that L meets Π at the point P.

(i) Demonstrate how the coordinates of P may be obtained from the system of equations in part **(b)**. *(2 marks)*

(ii) Hence determine the coordinates of P. *(2 marks)*

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7 The transformation T is represented by the matrix $\mathbf{M}$ with diagonalised form

$$\mathbf{M} = \mathbf{U}\,\mathbf{D}\,\mathbf{U}^{-1}$$

where $\mathbf{U} = \begin{bmatrix} 4 & -1 \\ 1 & 3 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 27 & 0 \\ 0 & 1 \end{bmatrix}$.

(a) (i) State the eigenvalues, and corresponding eigenvectors, of $\mathbf{M}$. *(4 marks)*

(ii) Find a cartesian equation for the line of invariant points of T. *(2 marks)*

Turn over ▶

(b) Write down $\mathbf{U}^{-1}$, and hence find the matrix $\mathbf{M}$ in the form

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

where a, b, c and d are integers. *(5 marks)*

(c) By finding the element in the first row, first column position of $\mathbf{M}^n$, prove that

$$4 \times 3^{3n+1} + 1$$

is a multiple of 13 for all positive integers n. *(5 marks)*

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8 The matrix $\begin{bmatrix} 12 & 16 \\ -9 & 36 \end{bmatrix}$ represents the transformation which is the composition, in either order, of the two plane transformations

E: an enlargement, centre O and scale factor k $(k > 0)$

and

S: a shear parallel to the line l which passes through O

Show that $k = 24$ and find a cartesian equation for l. *(7 marks)*

END OF QUESTIONS